Using repeating patterns to explore functional thinking

ELIZABETH WARREN and **TOM COOPER** lead us through a series of teaching activities designed to develop young children's algebraic thinking.

This paper continues the exploration of ways that we can reformulate common mathematical activities in the primary school to provide stronger bridges for thinking algebraically. The focus is not on introducing formal algebraic notation in the primary context, but rather to look at common activities through new lenses, that is, lenses that support the growth of algebraic reasoning. It seems that, as Malara and Navarra (2003) argued, classroom activities in the early years focus on mathematical products rather than on mathematical processes. Strings of numbers and operations in arithmetic are considered procedures for arriving at answers (Kieran, 1990). Traditionally, primary schools place minimal emphasis upon relations and transformations as objects of study. In our research we have found the young children can engage in conversations about equivalence and equations (Warren & Cooper, 2005a) and functional thinking (Warren & Cooper, 2005b). Fundamental to relations and transformations is the concept of the function, that is, how the value of certain quantities relate to the value of other quantities (Chazan, 1996), or how values are changed or mapped to other quantities, referred to in the literature as co-variational thinking. This paper reports on some recent classroom teaching that attempts to examine repeating patterns and use children's understandings of repeating pattern to begin to explore concepts related to functional thinking.

There are two predominant types of patterns that children explore in the early years: *repeating patterns* and *growing patterns*. Commonly, these patterns are used to find generalisations within the elements themselves: What comes next? Which part is repeating? Which part is missing? This activity is commonly referred to as pattern finding in a single variation data set (Blanton & Kaput, 2004). But repeating patterns and growing patterns can also lead to the early development of functional thinking, that is, relationships between two data sets. This paper presents a suggested sequence for investigating repeating patterns and extending these investigations to include activities and questions that specifically assist young children to begin to develop functional thinking.

Repeating patterns

Repeating patterns are patterns where a group of elements repeat themselves as the pattern extends. Some examples of repeating patterns are:

ABABABAB…

AABAABAABAABAAB…

ABCCABCC…

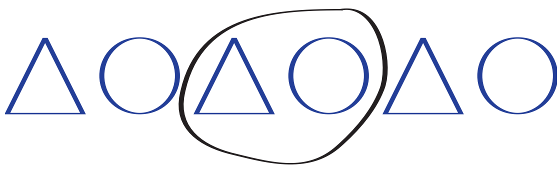
As we consider these patterns we need to present them in a variety of different modes. For example, the ABABAB pattern can be represented with actions (e.g., stand, sit, stand, sit, stand, sit, stand, sit), as sounds (drum beat, cymbal, drum beat cymbal), as geometric shapes (e.g.,▼●▼●▼●), and as feel (e.g., rough, smooth, rough, smooth, rough, smooth). When we are exploring repeating patterns there is a sequence that young children progress through. The next section delineates the suggested sequence, using an ABABAB pattern with geometric shapes to illustrate each step in the sequence.

Sequence for exploring repeating patterns

1. *Copying the pattern*
   Create the following patterns on the floor and encourage children to copy the pattern using triangles and circles.

2. *Continuing the pattern*
   In this instance ensure that young children realise repeating patterns can continue in both directions. Make the above pattern on the floor and ask the children. "What shape comes after the circle? What shape comes before the triangle." Get them to extend the pattern in both directions.

Number is an example of a pattern that extends in both directions.



3. *Identifying the repeating element*
   Say the pattern out loud (triangle, circle, triangle, circle, triangle, circle) and ask them to identify the part you are repeating. With a piece of wool, ask them to put a circle around the repeating part.

4. *Completing the pattern*
   Create a repeating pattern and encourage children to continue the pattern and identify the repeating part. With the children's eyes closed, remove some elements of the pattern. Ask the

children to open their eyes and tell you what shapes you have removed.

5. *Creating a pattern*
Encourage children to create their own repeating patterns. Ask, "Why is it a repeating pattern? Show me the repeating part? How would you continue your pattern?"

6. *Translating a pattern to a different medium*
This process is crucial to mathematics at all levels. It assists young children to develop the process of linking various representations and seeing the commonalities and differences between each. The differences are usually surface differences (e.g., clap, jump, clap, jump instead of triangle, circle, triangle, circle) and the commonalities are the structural aspects of mathematics (e.g., a pattern consisting of two elements and each element occurring once before it is repeated).

The above illustrates many of the common activities that typically occur in many early years classrooms. The next section delineates how we utilised this thinking to commence assisting young children to reason functionally and relationally. These ideas were tried in two classrooms, each with 25 children with an average age of 9 years and 6 months. In both instances the authors taught the lessons. The initial stage of the first lesson involved progressing through the above sequence, with an objective of ensuring that all children had a conceptual understanding of a repeating pattern. These children experienced little difficulty with this component of the lesson.

Viewing repeating patterns through new lenses

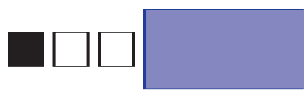
The aim of the next section was to look at these simple repeating patterns through new lenses, in ways that not only supported generalisation, but also provided opportunities to discuss the idea of function and proportion in the early years. The following lesson exemplifies the types of activities and questions that supported the development of this thinking. In this instance the simple ABBABBABB pattern was used and represented with red and green tiles, cut from sheets of foam. The children worked in pairs with each pair having their own set of tiles. The sequence was as follows:

Representing the pattern as a table of values

On board create the pattern

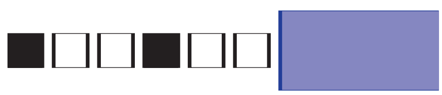
The children were asked to recreate this pattern with their own tiles.

With a card cover the pattern so that the first repeating part / the first set only is visible.



Ask, "How many red tiles are there? How many green tiles are there? How many tiles altogether?" Record the data in a table.

Uncover the pattern to show two sets of tiles.



Ask, "How many red tiles are there? How many green tiles are there? How many tiles altogether" Record the data on a table. Continue this sequence

until there are five sets visible, each time asking, "How many red tiles are there? How many green tiles are there? How many tiles altogether?". Record the data in the table.

No. of sets	No. of red squares	No. of green squares	Total number of squares
1	1	2	3
2	2	4	6
3	3	6	9
4	4	8	12
5	5	10	15

The children were then asked to examine the table and talk about the patterns that they saw in the table.

Previous research has indicated that children commonly look down the table to find patterns, for example, they tend to say that the number of green squares is increasing by 2 or the total number of tiles is increasing by 3. We refer to this as single variational thinking or the additive strategy (Warren, 1996). But the higher level of thinking and the one that leads to functional thinking is the looking for patterns across the table, that is, identifying the relationship between pairs of data (co-variational thinking). Thus students were asked to find three patterns down the table and three patterns across the table.

Some examples of children's responses were:

- *Down the table — sequential thinking or single variational thinking*
  "The reds are increasing by 1."
  "The greens are increasing by 2."
  "The total is increasing by 3."
- *Across the table — relational thinking or co-variational thinking*
  "The number of sets is the same as the number of reds."
  "There are twice as many greens as reds or half as many reds as greens."
  "The total is three times the number of sets."

We found that by restricting the number of each type of response to three, children were forced to look across the table as initially they kept sharing patterns down the table.

Generalising their thinking

The aim of this section was to encourage children to use their relational thinking generalisations to generate "uncountable" patterns in the table of values.

The following questions exemplify the thinking required for this.

If I had 10 red tiles, how many green tiles would I have?

If I had if I had 30 red tiles, how many green tiles would I have?

If I had 40 green tiles, how many red tiles would I have? How many sets of tiles would I have?

If I had 100 sets, how many red tiles would there be and how many green tiles would there be?

Most children completed these questions with a certain amount of ease. Most knew how to use the relational generalisations to find the answers (algebraic thinking) but some had difficulties computing the answers (arithmetic thinking). For this reason we grounded the explorations in number facts with which that they had had prior experience (the patterns of 2s, 3s and 5s) and chose numbers that we believed would be within their computational reach (e.g., 40 instead of 58).

A more challenging task was presented when they were given the total number of tiles and asked to work out how many green tiles, how many red tiles and how many sets. For example:

> Suppose I had 90 tiles: how many red tiles, how many green tiles?

For this thinking a common strategy was to find either the number of sets of tiles or number of red tiles by dividing the total number of tiles by 3 or by guessing which number you would multiply by 3 to give 90. They then doubled this number to give the number of green tiles.

We then moved to extending the table, placing different numbers in different columns and asked the children to complete the table. For example:

No. of sets	No. of red tiles	No. of green tiles	Total number of tiles
20			
	12		
			33
		40	

Generalising their thinking for an unknown number

Although this is not common practice in the primary school or even seen as appropriate in many classrooms, we decided to see how generally these children could think. Thus the following questions were asked:

> If I had an unknown number of sets of tiles, how would I work out how many red tiles, green tiles and the total number of tiles I had?

> If I have *n* repeats how many red tiles do I have and how many green tiles and how many in the total?

A number of children in these classes could correctly answer this question. Some of their responses were:

Helen: The number of red tiles is the same as the repeats, the green tiles double the repeats and the total triple.

Ben: For an unknown you double it for reds and three times it for total.

Jill: We have *n* [sounding a n sound] reds, *m* [sounding a m sound] because there is another arm on the *n* for number of green tiles and [sounding a long nnn sound] for the total because there are 2 arms on the *n*.

The above sequence was also repeated for AABBBAABBB pattern and for ABBBABBBABBB pattern.

Concluding comments

This lesson represents the first of a sequence of lessons that aimed to re-examine repeating patterns through different lenses. The underpinning belief we possessed at the beginning of the creation of this sequence of lessons was that children spend a great deal of time in the early years investigating repeating patterns. Our aim was to utilise this well-grounded thinking to explore more complex mathematical concepts. The question that motivated us in the development of these ideas was, "What is the pay off for more complex mathematical ideas such as functional thinking, proportionally reasoning, ratio and fractions?" The results of this first lesson show that not only are young children capable of thinking functionally, but also of representing this thinking in ways that we never

dreamed primary aged children could. Although pattern finding in single variable data sets is common in primary curricula, we are suggesting that primary grades' mathematics should extend this thinking to include functional thinking, that is, variation between data sets.

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